Name\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ Student number\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

**Assignment 4**

Determine rotation  of the bending beam shown at the support of the right end (use one element). The *x*-axis of the material coordinate system coincides with the neutral axis of the beam. Young’s modulus ** of the material and the second moment of cross-section  are constants. Use the virtual work density of the beam plane bending mode and a cubic approximation to the transverse displacement.

*L*

*Z*

*X*



1

2

*x*

*z*

**Solution template**

In the plane problem bending problem, when *x*-axis is chosen to coincide with the neutral axis, virtual work densitiesof the Bernoulli beam model are

 and .

Approximation is the first thing to be considered. The left end of the beam is clamped and the right end support allows only rotation. As only  is non-zero, approximation to  in terms of  simplifies into the form (see the formulae collection for the cubic beam bending approximation)

 

   so

 and .

When the approximation is substituted there, virtual work densities of the internal and external forces (external distributed force is constant) simplify to

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Integration over the domain  gives the virtual work expressions of the internal and external forces

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Principle of virtual work   and the fundamental lemma of variation calculus imply the solution

  

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